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EXCITATION OF PARAMETRIC OSCILLATIONS WITH RANDOM EXTERNAL SIGNALS

Currently, the main technical means of communication systems are receiving and transmitting radio devices, which in their structure contain various converters and amplifiers operating in nonlinear modes. In the transmission paths themselves, interference, delays, mismatches and parametric effects can occur, which leads to the appearance of small amplitude signals in the information channels. These factors have a significant impact on the characteristics of the telecommunications system.

Theoretical investigations of the influence of random signals (oscillations) on the operating modes of a nonlinear oscillatory system is an urgent task and aimed at expanding the possibilities of its use. In general, the analysis of random excitation of oscillations in nonlinear parametric systems is of interest for many branches of science and technology.

The goal of the work is to research the factors that influence the possibility of excitation of oscillations in nonlinear systems and possible modes of their operation in the presence of small in amplitude random signals.

The analysis of parametric systems with significant nonlinearity in the presence of random external signals. The most important cases of excitation of the inductive nonlinear system are considered. The relations allowing to control the parameters of the regenerate signal depending on the mode of excitation are got.

It is shown that in the case of a quasiharmonic input signal, the noise reduces the amplitude of the useful output signal. In the case of a broadband input signal (modeled by white noise), a limited response value is possible only for nonzero losses (the nonlinearity of the system is not enough for this).

Keywords: nonlinear system, parametric oscillations, excitation, random signal, inductive circuit, theoretical investigations.

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ЗБУДЖЕННЯ ПАРАМЕТРИЧНИХ КОЛИВАНЬ ПРИ ВИПАДКОВИХ ЗОВНІШНІХ СИГНАЛАХ

Теоретичні дослідження впливу випадкових сигналів (коливань) на режими роботи нелінійної коливальної системи є актуальним завданням і спрямовані на розширення можливостей її застосування. Взагалі, аналіз випадкового збудження коливань в нелінійних параметричних системах становить інтерес для багатьох галузей науки і техніки.

В роботі проведено аналіз параметричних систем з суттєвою нелінійністю при наявності випадкових зовнішніх впливів. Розглянуті найбільш важливі випадки збудження індуктивної нелінійної системи. Отримані співвідношення, які надають можливість керувати параметрами перетвореного сигналу в залежності від режиму збудження.

Ключові слова: нелінійна система, параметричні коливання, збудження, випадковий сигнал, индуктивный контур, теоретичні дослідження.

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ВОЗБУЖДЕНИЕ ПАРАМЕТРИЧЕСКИХ КОЛЕБАНИЙ ПРИ СЛУЧАЙНЫХ ВНЕШНИХ СИГНАЛАХ

Теоретические исследования влияния случайных сигналов (колебаний) на режимы работы нелинейной колебательной системы является актуальной задачей и направлены на расширение возможностей ее применения. Вообще, анализ случайного возбуждения колебаний в нелинейных параметрических системах представляет интерес для многих отраслей науки и техники.

В работе проведен анализ параметрических систем с существенной нелинейностью при наличии случайных внешних воздействий. Рассмотрены наиболее важные случаи возбуждения индуктивной нелинейной системы. Получены соотношения, позволяющие управлять параметрами преобразованного сигнала в зависимости от режима возбуждения.

Ключевые слова: нелинейная система, параметрические колебания, возбуждение, случайный сигнал, индуктивный контур, теоретические исследования.

Introduction

The excitation of parametric oscillations with random effects occurs in real electronic systems, in solids, mechanical systems, etc. The parametric converters themselves solve the problem of transforming oscillations in the broad sense of the word (generating, amplifying, measuring, dividing and multiplying, coding signals) with the necessary accuracy in a certain range of variation of their parameters. The error in the conversion depends on the amplitudes of the external signals, their time, frequency and other characteristics. In the general case inaccuracies are a random function of the distribution parameters describing the input signal. The characteristics of this function are usually specified and are a criterion in the design of radio engineering devices and telecommunications facilities.

Investigations of the influence of random signals (oscillations) on the operating modes of a nonlinear system are an urgent task and aimed at expanding its capabilities. The research of random excitation of oscillations in nonlinear systems is of interest for many branches of science and technology.

The goal of the work is to research the factors that influence the possibility of excitation of oscillations in nonlinear systems and possible modes of their operation in the presence of small in amplitude random signals.

Formulation of the problem

As a model for the research, we will consider an inductive parametric generator based on a resonant circuit with a nonlinear reactive element. It is important that

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research methods and the principles of operation of such devices can be used in researching other types of nonlinear systems [1].

Usually parametric converters are calculated for stationary modes and transformations of deterministic signals. The research of paths with an essential nonlinearity and random external influence in the theoretical plan is a very difficult task now, for the solution of which there are no common methods and they have not been sufficiently researched.

Main part

The inductive parametric generator, based on two magnetic cores with pumping (excitation) $W1$ windings, resonant $W2$ and losses $R1$, $R2$, respectively in the pump circuit and resonant as well as capacitance in the resonance circuit C, with symmetrical external action and approximation of the nonlinear magnetization characteristics

$$H = f(B) \tag{1}$$

as:

$$H = \alpha sh\beta B, \tag{2}$$

where: α, β – approximation coefficients;

B, H – instantaneous values of magnetic induction and magnetic field strength in the core,

are described by the following equations [2]:

$$\dot{x} + \gamma_1 sh\frac{x}{2}ch\frac{y}{2} = U_m\cos\tau; \tag{3}$$

$$\ddot{y} + \gamma_2\frac{1}{\omega}\frac{d}{d\tau}(ch\frac{x}{2}sh\frac{y}{2}) + \gamma_3 ch\frac{x}{2}sh\frac{y}{2} = f(\tau), \tag{4}$$

where: $\gamma_3 = \dfrac{\alpha\beta\ell}{SW_2^2\omega^2 C}$; $\gamma_1 = \dfrac{\alpha\beta\ell R_1}{SW_1^2\omega}$; $\gamma_2 = \dfrac{\alpha\beta\ell R_2}{SW_2^2\omega}$;

$\tau = \omega t$; $U_m = \dfrac{\beta U_m}{SW_1\omega}$;

U_m – amplitude of the pump voltage;

S – cross-sectional area of the core;

l – length of the midline magnetic field in the core;

ω – angular frequency of the pump voltage.

Here, x and y – reduced values of the voltages in the primary and secondary windings, the values of γ_1 and γ_3 define the relationship between the contours, and γ_2 describes the energy losses in the system. The right side in equation (3) is related to the action of the pump voltage, varying in harmonic order, the right side in equation (4) describes the action of an external signal (which can be random).

It is assumed that γ_1, $\gamma_3 << \gamma_2$. Then the response of the system to the signal $f(\tau)$ is determined by the equation:

$$\ddot{y} + \gamma_3 sh\frac{y}{2} + \gamma_2\frac{1}{\omega}\frac{d}{d\tau}(sh\frac{y}{2}) = f(\tau), \tag{5}$$

following from (4).

At first, we consider the case of a small signal $f(\tau) << \gamma_3$, which does not go into resonance with oscillations in the contour. These propositions enable us to linearize equation (3) with respect to y:

$$\ddot{y} + \gamma_3 y + \gamma_2\dot{y} = f(\tau). \tag{6}$$

We write the solution of (4) with the aid of the Green's function $G(\omega,t)$ of the corresponding homogeneous equation:

$$y(\tau) = \int_0^\tau G(\tau,t)f(t)dt. \tag{7}$$

The function $G(\omega,t)$ is expressed in terms of the linearly independent solutions $U_1(t)$, $U_2(t)$ of the homogeneous equation:

$$G(\tau,t) = \frac{U_1(t)U_2(\tau) - U_1(\tau)U_2(t)}{W(t)}, \tag{8}$$

where $W(t)$ – Wronskian determinant [3].

Expression (7) is convenient when analyzing a random effect on the system. It allows us to express all the characteristics of the response $y(\tau)$ in terms of the characteristics of the random signal $f(\tau)$. For example, the correlation function $<y(t_1)\ y(t_2)>$, which is of interest for applications, establishes a connection in the steady-state mode between the input and output signals in the time interval τ [4]:

$$< y(t_1)y(t_2) >= \int_0^{t_1}\int_0^{t_2} G(t_1,\tau_1)G(t_2,\tau_2) < f(\tau_2) > d\tau_1 d\tau_2\ . \tag{9}$$

Consider a lossless system ($\gamma_2 = 0$) in the absence of signals ($f(\tau) = 0$) at the initial time. Then the Green's function corresponding to it

$$G(x,t) = \sin\sqrt{\frac{\gamma_3}{2}}(x-t). \tag{10}$$

For example, consider the response of a system with Green's function (10) to compensation with random independent amplitudes A_n and deterministic phases ψ_n:

$$f(\tau) = \sum_n A_n\cos(n\omega\tau + \psi_n). \tag{11}$$

Let us show that even such a rough characteristic of the response $y(\tau)$, as the time-averaged value of the variance $<y^2(\tau)>$, contains information about the phases ψ_n, while the same characteristic for $f(\tau)$ does not contain this information. Carrying out the calculation by formula (9) using expressions (10) and (11), we obtain:

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T dt < y(t_1)y(t_2) >= \lim_{T\to\infty}\frac{1}{T}\int_0^T D_y(t)dt = \tag{12}$$

$$= \frac{1}{4}\sum_n < A_n^2 > (\frac{1}{(n\omega-\sqrt{b})^2} + \frac{1}{(n\omega-\sqrt{b})^2} - 2\frac{1}{n^2\omega^2 b}\cos 2\psi_n).$$

Let us investigate the resonant excitation of the circuit. It follows from equation (6) that the natural frequency of the system is $\omega_0 = \sqrt{\dfrac{\gamma_3}{2}}$. If ω_0 is close to an integer k, then one of the harmonics $f(\tau)$ causes a resonance in the circuit. We assume that the main mechanism limiting the amplitude of the oscillations is the nonlinearity of the term with the coefficient γ_3 in the expression (3) (that is, the losses proportional to γ_2 are sufficiently small). Without loss of generality, we can also assume that the resonance is due to the lowest harmonic in the perturbation $f(\tau)$. Neglecting the losses and taking into account the first two terms in the expansions for the function $sh\dfrac{y}{2}$, and also leaving only the resonance term in the signal $f(\tau)$, we obtain:

$$\ddot{y} + \frac{\gamma_3}{2}y + \frac{\gamma_3}{48}y^3 = A\cos(\tau + \psi). \tag{13}$$

These approximations are justified if the signal amplitude is $A \le 1$ (for $A = 0$ equation (13) is called the Duffing`s equation). We seek the solution of (13) in the form of a Fourier series:

$$y(\tau) = \sum_{k=0}^{\infty} B_k \cos k(\tau + \psi) . \quad (14)$$

Substituting relation (14) into expression (13) gives:

$$\sum_k (\frac{\gamma_3}{2} - K^2) B_k \cos(\tau + \psi) + \frac{\gamma_3}{48} (\sum_k B_k \cos k(\tau + \psi))^3 =$$
$$= A \cos(\tau + \psi). \quad (15)$$

We believe that $\left| \frac{\gamma_3}{2} - 1 \right| \le 1$. Therefore, the harmonic $B_1 >> B_n$ for all $n > 1$. Indeed, it follows from (15) that $B_n \sim A$ for $n > 1$, and $B_1 \sim A^{1/3} >> A$. Leaving only the terms proportional to cos (τ + ψ) in expression (15), we obtain:

$$(\frac{\gamma_3}{2} - 1) B_1 + \frac{\gamma_3}{64} B_1^3 = A. \quad (16)$$

Equation (13) has a solution for the case of exact resonance (for $\gamma_3 = 2$):

$$y(\tau) = 2(4A)^{\frac{1}{3}} \cos(\tau + \psi). \quad (17)$$

The solution (17) is valid for any realization of the random variables A and ψ, that is, the response properties of $y(\tau)$ as a random variable are completely determined by those for A and ψ. All other harmonics in $f(\tau)$ cause a small perturbation (moderately small A) of the solution (17).

Define the mathematical expectation $< y(\tau) >$ of the solution (17), assuming A to be a random variable distributed according to the normal law with parameters A_0 and σ. It is obvious that the mathematical expectation:

$$< y(\tau) >= y_0 \cos(\tau + \psi), \quad (18)$$

where: $y_0 = \frac{1}{\sqrt{2\pi}} A_0^{\frac{1}{3}} \int_0^{\infty} [(1 - \frac{\sigma}{A_0} z)^{\frac{1}{3}} + (1 + \frac{\sigma}{A_0} z)^{\frac{1}{3}}] \ell^{-\frac{z^2}{2}} dz,$

(σ – standard deviation). (19)

A direct calculation shows that $\frac{\partial y_0}{\partial \sigma} < 0$, i.e. for a given A_0, the noise (described by the dispersion $D = \sigma^2$) decreases the amplitude of the useful signal y_0, which is a manifestation of the nonlinearity of the considered system.

The example considered above demonstrated the effect of a quasiharmonic signal on a nonlinear system. The model of a random effective force with a wide spectrum is a Gaussian (normal) random and σ-correlated process, where the random variable $f(\tau)$ at each instant τ is distributed according to a Gaussian law, and the correlation function has the form [5]:

$$< f(t_1) f(t_2) >= 2D\sigma(t_1 - t_2). \quad (20)$$

Taking into account the fact that $\gamma_3 >> \gamma_2$, we leave in equation (6) linear with respect to the derivative $\dot{y}$, the response and the cubic terms along the response y the terms

$$\ddot{y} + \frac{\gamma_3}{2} y + \frac{\gamma_3}{48} y^3 + \frac{\gamma_2}{2} \dot{y} = f(\tau) . \quad (21)$$

We introduce the canonical variables $p = \dot{y}$ and y in expression (19). Then, taking into account the Hamiltonian function $H = \frac{p^2}{2} + \frac{\gamma_3}{4} y^2 + \frac{\gamma_3}{4 \cdot 48} y^4$, equation (21) can be written in the form of a Hamiltonian system of equations [6, 7]:

$$\dot{y} = \frac{dH}{dP}, \qquad \dot{p} = -\frac{dH}{dy} - \frac{\gamma_2}{2} p - f(\tau). \quad (22)$$

The probability distribution function P for $p(t)$ and $y(t)$ can be found from the Einstein-Fokker equation [7]:

$$\frac{dP}{dt} + \frac{dP}{dP}\frac{dH}{dy} - \frac{dP}{dy}\frac{dH}{dP} - \frac{\gamma_2}{2}\frac{d}{dP}(pP) = D\frac{d^2 P}{dP^2}. \quad (23)$$

The time-independent solution of equation (23) has the form:

$$P_{(p,y)} = const\ e^{-\frac{\gamma_2}{2D} H} \equiv$$
$$\equiv const \exp\left\{ -\frac{\gamma_2}{2D} [\frac{P^2}{2} + \frac{\gamma_3}{4} y^2 + \frac{\gamma_3}{4 \times 48} y^4] \right\}. \quad (24)$$

Here *const* is found from the normalization condition for the probability density. The expression (24) gives complete information about the statistical properties of equation (21). Thus, it is seen from expression (24) that the derivative $\dot{y}$ and the response y are statistically independent, since $P(p,y)$ is factorized: $P(p,y) = P_1(p,y) \odot P_2(p,y)$. The magnitude of the derivative $\dot{y}$ has a normal distribution, and of the response y – has no:

$$P_2(y) \sim exp\left\{ -\frac{\gamma_2}{D} [\frac{\gamma_3}{4} y^2 + \frac{\gamma_3 y^4}{4 \times 48}] \right\}. \quad (25)$$

It follows from (25) that $<y> = 0$, $<y^2> \ne 0$, i.e. the system performs random "oscillations" near the origin. Their amplitude increases with increasing force (proportional $\sqrt{D}$) and a decrease in friction (proportional to γ_2).

Thus, we considered the effect of a random signal without taking into account the effect of the pump on the resonant circuit. In the nonlinear mode, the characteristics of the random function-response differ substantially from the characteristics of the random function-signal. It is shown that in the case of a quasiharmonic input signal, the noise reduces the amplitude of the useful output signal. In the case of a broadband input signal (modeled by white noise), a limited response value is possible only for nonzero friction – the nonlinearity of the system is not enough for this. The resulting function $P(\dot{y}, y)$ – (24) describes the response of the system to white noise.

Let us investigate the transformation of a random signal by a parametrically excited system. The most interesting case is when the frequency of natural oscillations is $\omega^2{}_0 = \frac{\gamma_3}{2}$ of the order of unity; then the condition $\gamma_3 >> \gamma_1, \gamma_2$ implies:

$$\gamma_1, \gamma_2 << 1. \quad (26)$$

Neglecting the deviation of the response y from zero in equation (3), which is equivalent to neglecting the inverse action of the excited contour on the pump source, and expanding $sh\frac{x}{2}$ under the restriction by the cubic term of the series, we obtain:

$$\dot{x}+\frac{\gamma_3}{2}x=U_m\cos\tau-\frac{1}{48}\gamma_1x^3. \qquad (27)$$

If the last term on the right-hand side of expression (27) is not taken into account, then the last equation is easily solved:

$$x_0=\frac{4}{\gamma_1^2+4}U_m(\sin\tau+\frac{\gamma_1}{2}\cos\tau)\approx U_m(\sin\tau+\frac{\gamma_1}{2}\cos\tau)\,;(28a)$$

$$x_0=-\frac{4}{\gamma_1^2+4}U_m(\cos\tau-\frac{\gamma_1}{2}\sin\tau)\approx -U_m(\cos\tau-\frac{\gamma_1}{2}\sin\tau)\,.(28b)$$

The passage to the last expression in (28b) is made using the inequality (26). We find the correction to (28b) due to the nonlinearity of equation (27). In the first approximation in γ_1 we have:

$$x=x_0(\tau)+x_1(\tau),$$

$$x_1=-\frac{1}{48}\gamma_1U_m^3\frac{1}{4}(\cos3\tau+3\cos\tau),$$

or $$x_1=\frac{\gamma_1}{64}U_m^3\sin\tau+\frac{\gamma_1}{576}U_m^3\sin3\tau. \qquad (29)$$

Comparison of expressions (28) and (29) shows that

$$\left|\frac{x_1}{x_0}\right|\sim\frac{\gamma_1}{64}U_m^2. \qquad (30)$$

Thus, if

$$U_m\le 1, \qquad (31)$$

then, according to approximation (30), the ratio $\left|\frac{x_1}{x_0}\right|$ in addition to the small parameter γ_1, contains one more small parameter $\left(\frac{U_m}{8}\right)^2$. In what follows, condition (31) is assumed to be satisfied, and the solution $x(\tau)$ is the same as x_0:

$$x_0=-U_m\cos(\tau+\varphi_1);\quad \varphi_1=arctg\frac{\gamma_1}{\sqrt{4+\gamma_1^2}}\approx\frac{\gamma_1}{2}. \qquad (32)$$

Now we simplify expression (4)

$$\gamma_2\frac{1}{\omega}\frac{d}{d\tau}(sh\frac{y}{2}ch\frac{x}{2})+\frac{1}{2}\gamma_2\dot{y}+\frac{1}{8}\gamma_2x\dot{x}y=0\,. \qquad (33)$$

In expression (33), the first term of the expansion $sh\frac{y}{2}$ and the first two terms in the expansion $ch\frac{x}{2}$ are taken into account. Taking into account expressions (32) and (33), we obtain:

$$\ddot{y}+\left\{\frac{1}{2}\gamma_3+\frac{\gamma_3U_m^2}{16}\cos^2(\tau+\varphi_1)+\frac{\gamma_2U_m^2}{16}\sin(2\tau+2\varphi_1)\right\}y+$$

$$+\frac{1}{2}\gamma_2\dot{y}=f(\tau). \qquad (34)$$

It is convenient to rewrite equation (34) by introducing the following designations

$$\varphi=\varphi_1+arctg\frac{\gamma_2}{16\sqrt{(\frac{\gamma_3}{32})^2+(\frac{\gamma_2}{16})^2}}\approx\frac{\gamma_1}{2}+\frac{2\gamma_2}{\gamma_3};$$

$$2\alpha=\frac{1}{2}\gamma_2;\ \ \omega_0^2=\frac{1}{2}\gamma_3;\ \ m=U_m\sqrt{\left(\frac{1}{16}\right)^2+(\frac{\gamma_2}{8\gamma_3})}\approx\frac{U_m}{16}\,. \qquad (35)$$

In the new designations we obtain:

$$\ddot{y}+2\alpha\dot{y}+\omega_0^2(1+m\cos(2\tau+\varphi))y=f(\tau). \qquad (36)$$

The frequency $\omega_н$, as can be seen from equation (36), in the dimensionless variables is $\omega_н=2$. The pump amplitude m, as follows from expressions (31) and (35), is small:

$$m<<1. \qquad (37)$$

The solution of equation (36) can be written in terms of the response function $k(\omega,\tau)$ by the periodic force $f(t)\sim e^{i\omega\tau}$.

The presence of a small parameter m makes it possible to develop an approximate calculation procedure for $k(\omega,\tau)$. Using the expansion

$$k(\omega,\tau)=\sum_n k_n(\omega)e^{i\omega\tau} \qquad (38)$$

when solving equation

$$\ddot{y}+2\alpha\dot{y}+\omega_{02}(1+m\cos(2\tau+\varphi))y=e^{i\omega\tau} \qquad (39)$$

we obtain the following system of coupled equations for the coefficients $k_n(\omega)$ of the Fourier expansion (38):

$$[\omega_0^2-(\omega-2n)^2+2i\alpha(\omega-2n)]k_n(\omega)+\frac{m\omega_0^2}{2}[e^{-i\varphi}k_{n-1}(\omega)]=$$

$$=\delta_{n_0}\,, \qquad (40)$$

where: $n=0;\pm1;\pm2;\ldots;$

δ_{n0} – Koneker symbol.

Let the condition:

$$\left|(\omega+2n+\omega_0)(\omega-2n+\omega_0)\right|>>\frac{n\omega_0^2}{2};\quad n=\pm1,\ldots \qquad (41)$$

(In the case when condition (41) is violated, they speak of resonance at the combination frequency). Then, as follows from expression (40), the ratio $\frac{k_n}{k_{n-1}}\sim\frac{k_{-n}}{k_{-n+1}}\sim m<<1$ (here $n>1$), and the system (40) can be solved by the method of successive approximations.

We write out the first three equations of system (40), which allow us to determine the coefficients k_0, k_1, k_{-1} up to terms proportional to m.

$n=0;$

$$(\omega_0^2-\omega^2+2i\alpha\omega)k_0(\omega)+\frac{m\omega_0^2}{2}(e^{-i\varphi}k_1(\varphi)+e^{i\varphi}k_1(\varphi))=1; \qquad (42)$$

$n=1;$

$$(\omega_0^2-(\omega-2)^2+2i\alpha(\omega-2))k_1(\omega)+\frac{m\omega_0^2}{2}(e^{-i\varphi}k_0(\varphi)+e^{i\varphi}k_2(\varphi))=0;$$

$n=-1;$

$$(\omega_0^2-(\omega+2)^2+2i\alpha(\omega+2))k_1(\omega)+\frac{m\omega_0^2}{2}(e^{-i\varphi}k_2(\varphi)+e^{i\varphi}k_0(\varphi))=0.$$

Thus, in this paper, for an inductive nonlinear parametric contour with random external action, equations were obtained analytically that allow us to determine the coefficients k_0, k_1, k_{-1} in the Fourier expansion up to terms proportional to the small parameter m.

Conclusion

1. The effect of a random signal on a nonlinear system is considered and it is shown that in the nonlinear

mode of operation the characteristics of the random function-response differ substantially from the characteristics of the random signal.

2. It is shown that in the case of a quasiharmonic input signal, the noise reduces the amplitude of the useful output signal. In the case of a broadband input signal (modeled by white noise), a limited response value is possible only for nonzero losses (the nonlinearity of the system is not enough for this).

3. The transformation of a random signal in a nonlinear system with parametric excitation is researched, when the frequency of its own oscillations is lower than the frequency of the main signal. The equations that allow us to determine the coefficients k_0, k_1, k_{-1} in the Fourier expansion are obtained analytically to within terms proportional to the small parameter m.

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